ST. BRIGID'S COLLEGE

Mathematics Specialist Units 3 & 4
Test 4 2017

Section 1 Calculator Free

Vector Calculus in Two Dimensions

STUDENT'S NAME: _____

DATE: Thursday 18th May **TIME:** 25 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet.

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

If curvilinear motion is modelled by position vector $\boldsymbol{r}(t) = (t-1)\boldsymbol{i} + t^2\boldsymbol{j}$, $t \geq 0$, then:

(a) Sketch the path of the motion [3]

(b) Determine the Cartesian equation of the motion. [4]

2. (15 marks)

The curvilinear motion of an object is defined by $\boldsymbol{r}(t) = (t+1)\boldsymbol{i} + (4t - t^2)\boldsymbol{j}, t \geq 0$. Determine:

(a) The initial position. [2]

(b) The distance from the origin when $t = 5$. [3]

(c) The time when the particle lies on the x-axis. [3]

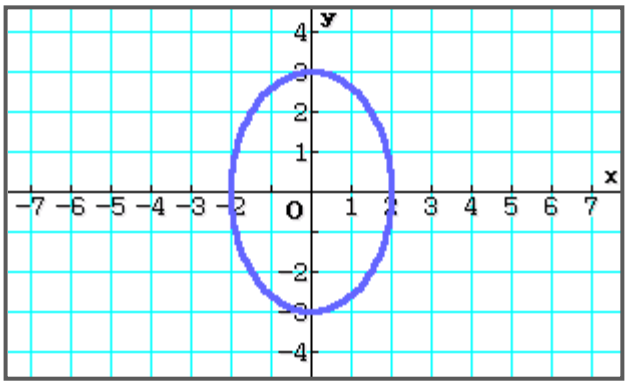
(d) The maximum distance the particle is from the x-axis. [4]

(e) The initial speed. [3]

3. (8 marks)

Consider the following screen capture of elliptical motion, modelled with position vector:

$$\boldsymbol{r}(t) = a\sin(t)\,\boldsymbol{i} + b\cos(t)\,\boldsymbol{j},\ 0 \le t < 2\pi$$



(a) State the values of a and b, where $a > 0$ and $b > 0$. [2]

(b) Draw on the diagram the initial position vector and indicate with an arrow the direction of the motion. [2]

(c) Rewrite the position vector, $\boldsymbol{r}(t)$, such that the motion starts at position $(0, -3)$, travels clockwise with half the speed, to complete one full ellipse. [4]

End of Section 1 Questions

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Section 2 Calculator Assumed

Vector Calculus in Two Dimensions

STUDENT'S NAME: _____

DATE: Thursday 18th May **TIME:** 25 minutes **MARKS**: 30

INSTRUCTIONS:

Standard Items: Pens, pencils, pencil sharper, eraser, correction fluid/tape, ruler, highlighters, Formula Sheet retained from Section 1.

Special Items: Drawing instruments, templates, three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment).

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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4. (7 marks)

A particle is moving around a circle of radius r such that its position vector, $\boldsymbol{r}(t)$, is given by:

$$\boldsymbol{r}(t) = r\cos(\omega t)\,\boldsymbol{i} + r\sin(\omega t)\,\boldsymbol{j}$$

(a) Determine the velocity vector $\boldsymbol{v}(t)$. [2]

(b) Show, using the scalar (dot) product property: $|\boldsymbol{v}(t)|^2 = \boldsymbol{v}(t)\bullet\boldsymbol{v}(t)$, that the speed is $v = \omega r$ [3]

(c) Calculate the scalar (dot) product: $\boldsymbol{v}(t)\bullet\boldsymbol{r}(t)$ [2]

5. (17 marks)

A stone is thrown from a veranda 12 m above horizontal ground, with an initial velocity of 25 ms^{-1} at a 55˚ angle of elevation. Determine:

(a) Determine the position vector of the stone. [5]

(b) The time taken for the stone to strike the ground. [2]

(c) The horizontal distance travelled by the stone from its point of projection to where it meets the ground. [2]

(d) The greatest height above the ground reached during flight. [3]

(e) The distance between the point of projection and where the stone hits the ground. [2]

(f) The total distance travelled by the stone. [3]

6. (6 marks)

It can be shown, by extending the argument used in Question 4, that uniform circular motion has acceleration: $\boldsymbol{a}(t) = -\omega^2 \boldsymbol{r}(t)$

(a) Explain in words the implication of the above result. [3]

A body attached to a string 4 m long, moves on a smooth horizontal plane surface at a speed of 8 ms^{-1}.

(b) What is the magnitude of the body's acceleration? [3]

End of Questions